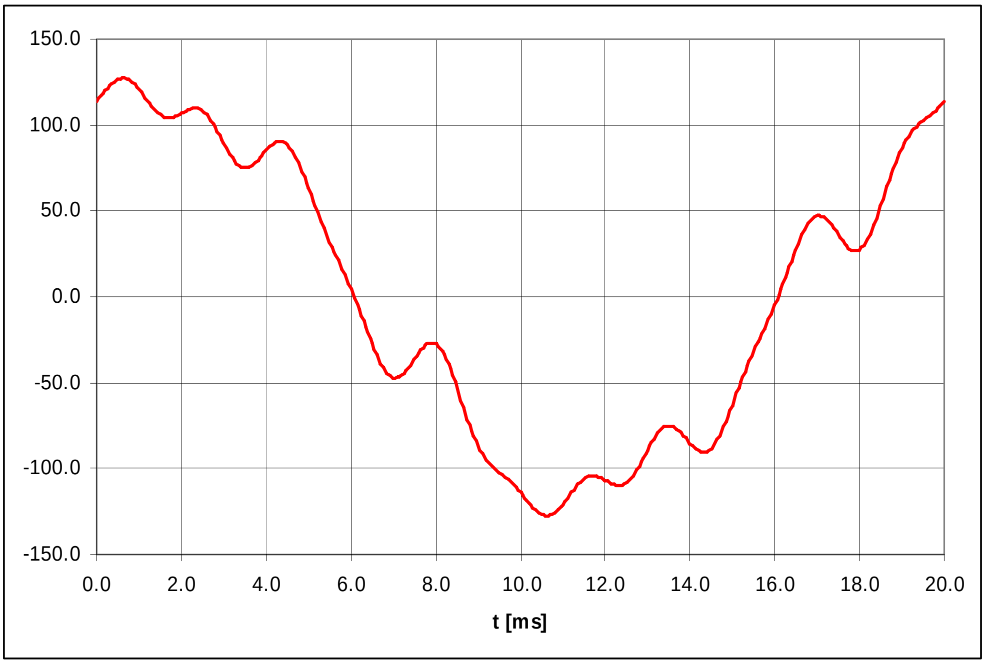
1 Harmonic Load Characteristic

A harmonic load has the following current waveform:



and the corresponding data must be inputted in DIgSILENT PowerFactory.

The waveform is periodic with a period of 50 Hz. The frequency of 50 Hz is therefore considered the fundamental one.

A waveform, as a general rule, can therefore be represented as:

$$i(t)=\sum_{h=1}^{N}\left(a_h\cdot\cos(h\omega t)+b_h\cdot\sin(h\omega t)\right)=\sum_{h=1}^{N}c_h\cdot\cos(h\omega t+\varphi_h) \qquad [1],$$

and where $\omega = 2\pi\cdot f = 2\pi\cdot 50$ Hz.

The values of the modules c_h and of the phase φ_h can be easily found from a_h and b_h:

$$c_h\cdot\cos(h\omega t+\varphi_h)=c_h\cdot\cos(\varphi_h)\cdot\cos(h\omega t)-c_h\cdot\sin(\varphi_h)\cdot\sin(h\omega t);$$

therefore:

$$\begin{aligned}a_h &= +c_h\cdot\cos(\varphi_h)\\ b_h &= -c_h\cdot\sin(\varphi_h)\end{aligned};$$

thus:

$$c_h=\sqrt{a_h^2+b_h^2} \qquad [2.1];$$

$$\varphi_h=\arctan\left(\frac{-b_h}{a_h}\right) \qquad [2.2].$$

The FFT of this waveform shows that it contains only 4 harmonics: h = 1, 5, 7, 11, where h is the harmonic order. The values of the harmonic coefficients are:

h	a_h	b_h	c_h	φ_h [rad]	φ_h [deg]
1	105	48	115.45	-0.428778	-24.5672
5	12	11	16.28	-0.741947	-42.5104
7	3	-9	9.49	1.249046	71.5651
11	-6	5	7.81	-2.446854	-140.1944

In order to input the data in PowerFactory, the amplitudes c_h must be espressed in % of c_1 and the phase angles must be referred to the phase angle of the 1st harmonics.
The conversion of the amplitudes in % of c_1 is very easy:

$$c_{h,\%} = 100\frac{c_h}{c_1} \qquad [3]$$

while the change of reference angle must be considered more carefully.
The simple operation of subtracting φ_1 to each φ_h is wrong.
The correct operation corresponds to change the time origin, in such a way that the 1st harmonics becomes a pure cosine (φ_1 = 0). Therefore, within the new origin a new time quantity can be defined:

$$\cos(\omega\hat{t}) = \cos(\omega t + \varphi_1) \qquad [4.0];$$

therefore:

$$\hat{t} = t + \frac{\varphi_1}{\omega} \qquad [4.1];$$

and also:

$$t = \hat{t} - \frac{\varphi_1}{\omega} \qquad [4.2].$$

As a consequence, all the other harmonics can be written as:

$$c_h \cdot \cos(h\omega t + \varphi_h) = c_h \cdot \cos\left(h\omega\left(\hat{t} - \frac{\varphi_1}{\omega}\right) + \varphi_h\right) = c_h \cdot \cos(h\omega\hat{t} - h\varphi_1 + \varphi_h) = c_h \cdot \cos(h\omega\hat{t} + \hat{\varphi}_h) \qquad [4.3],$$

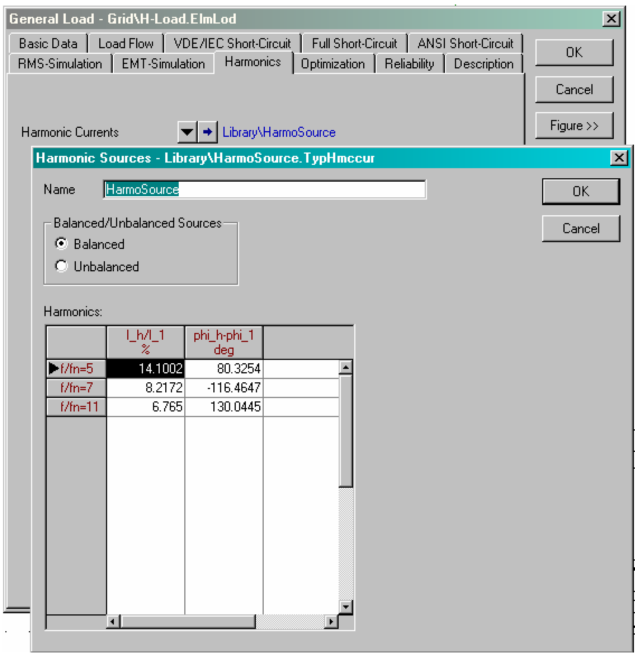
where:

$$\hat{\varphi}_h = +\varphi_h - h\varphi_1 \qquad [5].$$

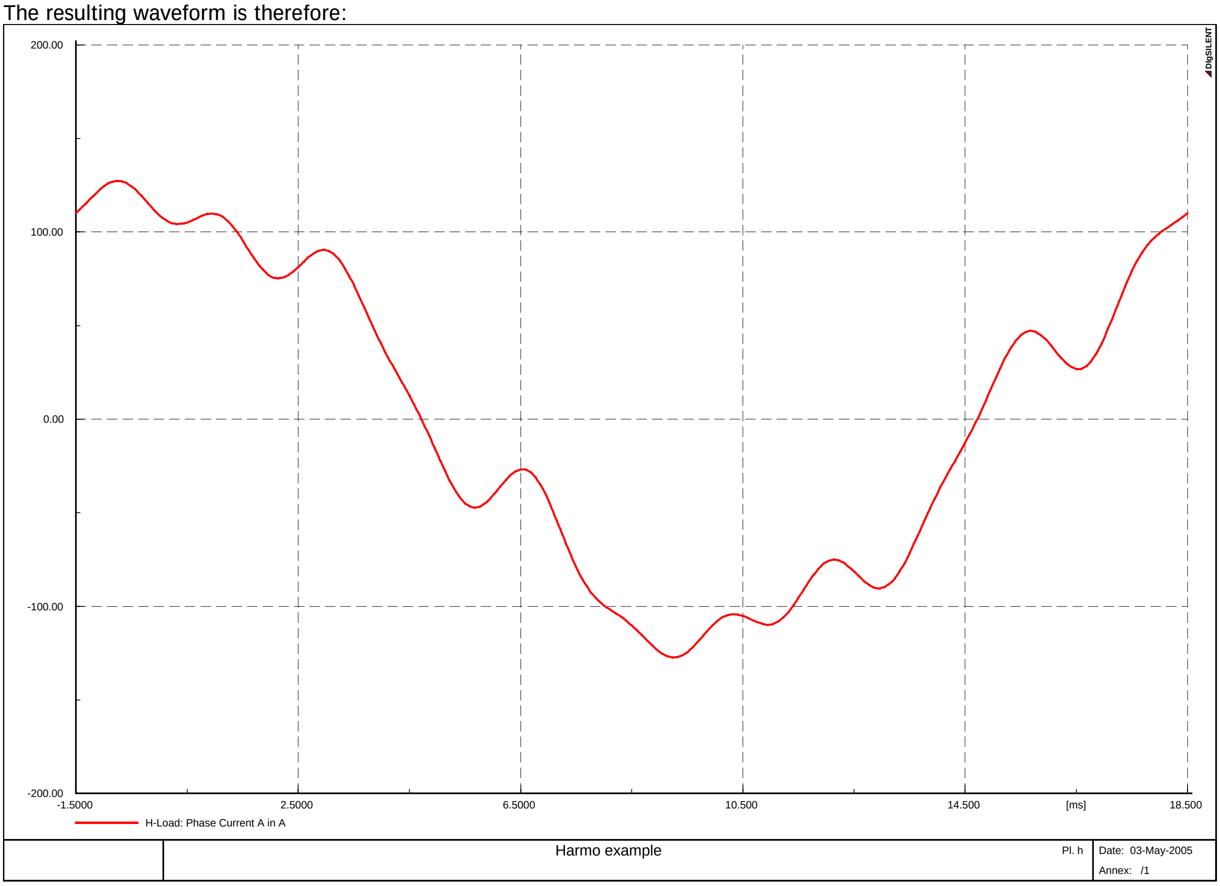
Using [3] and [5], the data to be inputted are (the angle values are reported in the interval -180°÷180° by adding k·360°):

h	c_h [%]	φ_h [deg], referred to 1st harmonics
1	100.0000	0.0000
5	14.1002	80.3254
7	8.2172	-116.4647
11	6.7650	130.0445

data input in the load mask of DigSILENT P.F.:



The resulting waveform is therefore:



that is exactly the required one (just shifted of a few milliseconds).