

# EXPLOITING MULTIPLE TIME SCALE PROPERTIES FOR FAST SIMULATION ALGORITHMS

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## ABSTRACT

Adaptive step size algorithms are presented making use of the multiple time scale character of power systems. The paper analyzes methods using multiple step sizes simultaneously (multirate methods) and methods adapting the integration step size in time. The resulting algorithm covers transients of all time ranges, from electromagnetic to electromechanical phenomena.

## 1. INTRODUCTION

Transients in power systems are characterized by eigenvalues which are lying in different order of magnitude. Hence, the electrical power system is a *multiple time scale* system and the resulting differential equations are stiff. Simulation algorithms can profit by the following two aspects of multiple time scale behavior:

- The damping of fast transients is (normally) much higher than for slow transients.
- State variables can be chosen in such a way that only a subset of them is influenced by fast movements. The overall system can then be decomposed into different subsystems characterizing transients ranging in different time scales.

In this paper, the power system is seen as a two scale system characterized by the following types of transients:

- Electromechanical transients, slow subsystem
- Electromagnetic transients, fast subsystem

In commonly used simulation tools for transient stability, a *large time scale* approximation is used by considering a steady state representation of the electrical grid.

Voltages and currents are here expressed by complex phasors. Capacitances and inductivities are then described by their complex steady state equations.

But with this representation, it's not possible to analyze electromagnetic transients, even decaying DC-components of currents cannot be taken into account.

EMT programs (e.g. [1]) however, would generally be able to cover the whole range of transients in power systems, but because the instantaneous values of voltages and currents are sinusoidal in steady state, the simulation step size must remain so small, that longer term transients cannot be analyzed in a reasonable amount of calculation time.

Existing solutions for these problems are:

- Exchanging the dynamic network model by a steady state model after fast transients have decayed. But the sudden change of network models can cause parasitic excitations of the overall system because the network models never match completely, even if fast transients have decayed.
- Describing the electrical grid equations in a rotating reference frame leading to a constant course of voltages and currents in a balanced steady state. The integration step size can here be increased up to several milliseconds [2].

But this method is only efficient in balanced situations and is not able to deal with zero sequence components at all.

In the center of interest of this paper are transients, ranging from about 100 microseconds up to one minute, particularly in unbalanced system configurations.

Our approach consists of

- Using an individual integration step size for each subsystem: *Multirate algorithm*
- Modifying existing integration methods in such a way, that the sinusoidal steady state of the electrical grid is always represented correctly, independent of the integration step size, including unbalanced systems. Such a method has been presented in [3] and is further developed in this paper.

The first part of the paper describes the general structure of a multirate algorithm including a discussion of problems associated with it and their solutions.

In the second part we present an adaptive step size algorithm based on the integration method described in [3].

Finally, examples demonstrate the precision and efficiency of the described simulation methods.

## 2. MULTIPLE TIME SCALE SYSTEMS

If a subset of state variables is only weakly influenced by fast transients, the overall system can be split into subsystems characterizing transients of different time ranges. An analytical method to perform this decomposition is the *singular perturbation analysis* [4].

The results of a two time scale analysis can be resumed as follows:

- Small time scale analysis: Large eigenvalues are characterized only by the fast subsystem. State variables of the slow subsystem can be considered as input variables to the fast subsystem.
- Large time scale analysis: Considering that fast transients have decayed, slow transients can be calculated by considering the slow subsystem together with the steady state behavior of the fast subsystem.

Describing the electrical grid in a steady reference frame or original *abc*-phase coordinates and the slow subsystem in a rotating reference frame, the transformation of reference frames has additionally be taken into account.

Consequently, the overall system can be defined by the following set of differential equations:

$$\begin{aligned} \dot{x}_s &= \mathbf{f}(x_s, \mathbf{T}^{-1}x_f) \\ 0 &= \mathbf{g}(x_f, \dot{x}_f, \mathbf{T}x_s) \end{aligned} \quad (1)$$

The state variables  $x_s$  together with the function  $\mathbf{f}$  are building the slow subsystem. In power systems, the variables  $x_s$  can be rotor fluxes, machine speed, angles but also controller state variables.

Because the electrical grid is normally described by an implicit system of differential equations, the implicit formulation has been chosen for the fast subsystem. The state variables  $x_f$  are e.g. the instantaneous values of line currents, voltages across capacitances or stator fluxes of machines.

The transformation-matrix  $\mathbf{T}$  describes the transformation between *abc* co-ordinates and a rotating reference frame (*dq0*). In case of a three phase variable the transformation can be described as follows:

$$u_{abc} = \mathbf{T}_{3 \times 3} u_{dq0} \quad (2)$$

with

$$\mathbf{T}_{3 \times 3} = \begin{bmatrix} \cos \vartheta & -\sin \vartheta & 1 \\ \cos(\vartheta - \frac{2\pi}{3}) & -\sin(\vartheta - \frac{2\pi}{3}) & 1 \\ \cos(\vartheta - \frac{4\pi}{3}) & -\sin(\vartheta - \frac{4\pi}{3}) & 1 \end{bmatrix} \quad (3)$$

The angle  $\vartheta$  is defined by:

$$\vartheta = \omega_r t$$

The angular frequency  $\omega_r$  is the frequency of the rotating reference frame.

The complete transformation matrix  $\mathbf{T}$  has a block diagonal structure in which each  $3 \times 3$  block corresponds to the transformation matrix of (3).

## 3. MULTIRATE ALGORITHM

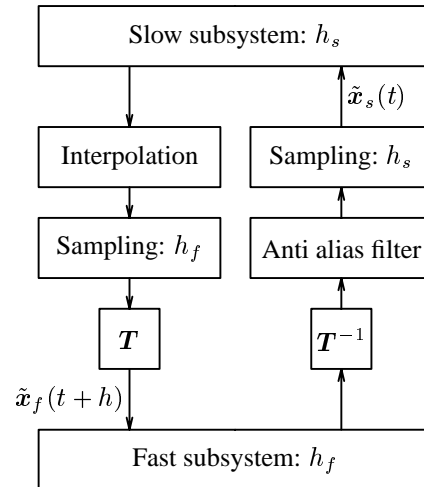


Figure 1: Coupling of subsystems in a multi-rate algorithm

Multirate algorithms are very useful for real time applications, because they reduce the calculation time *per time step* considerably.

Because state variables of the slow subsystem are almost free of fast transients, a high integration step size can be used for the slow subsystem compared to the fast subsystem. Implementations of multirate algorithms are described in [5] or in [3].

Using different rates, the variables being exchanged have to be resampled. Therefore, interfaces are required performing the necessary interface operations.

The structure of a multirate algorithm, including interpolation and anti alias filtering is shown in fig. 1.

Particularly the low-pass filter which has to prevent aliasing caused by high frequencies in  $x_f$  is introducing additional errors because causal low pass filters, like moving average filters, always have a lagging phase characteristic.

Because interpolation is used for feeding slow state variables to the fast subsystem, the next value of a slow state variable  $x_s(t + h_s)$  must always be known before the fast subsystem can be evaluated. Therefore, the differential equations describing the slow subsystem must be solved by an *explicit* numerical integration algorithm.

## 4. ADAPTIVE STEP SIZE

In algorithms using a three phase instantaneous value representation for modelling the electrical grid, the integration step size is very limited, even if fast transients have decayed.

The problem here is the precision of the numerical integration method used for discretizing the fast subsystem. Because of the transformation matrix  $\mathbf{T}$  in (1), the steady state

of the fast subsystem is characterized by a sinusoidal course of all fast state variables  $x_f$  to which the integration step size has to be adapted.

The next sections present a method which allows to increase the integration step size up to values which are commonly used for analyzing electromechanical transients ( $h \approx \frac{1}{2f_n}$ ).

#### 4.1 Linear Integration Methods

Linear integration methods are widely used in power system simulation programs for analyzing electromagnetic transients (fast subsystem). Particularly the trapezoidal rule which is defined by

$$x(t+h) = x(t) + b_0\dot{x}(t) + b_1\dot{x}(t+h) \quad (4)$$

with  $b_0 = b_1 = h/2$  is used in many programs (e.g. [1]). The local truncation error, an expression describing the error introduced by one simulation step, can be used for estimating the precision of the numerical method. For numerical integration methods according to (4), the local truncation error is defined as follows (see e.g. [6]):

$$\varepsilon_l = x(t+h) - x(t) - b_0\dot{x}(t) - b_1\dot{x}(t+h) \quad (5)$$

For  $b_0 = b_1 = h/2$ , equation (5) is:

$$\varepsilon_l = x(t+h) - x(t) - \frac{h}{2}\dot{x}(t) - \frac{h}{2}\dot{x}(t+h) \quad (6)$$

Approximating  $x(t)$  by a Taylor series leads to an approximated value for  $\varepsilon_l$ :

$$\varepsilon_l \approx -\frac{h^3}{12}\ddot{x}(t) \quad (7)$$

For evaluating (7) during a simulation, the calculation of  $\ddot{x}(t)$  needs to be performed at each time step. This could be done by approximating the derivatives with difference formulas but then, the evaluation of more history values is required and additional imprecisions are introduced.

A more convenient way of estimating  $\varepsilon_l$  is the use of a predictor formula of the same order as the implicit integration method.

A predictor formula of the same order as the trapezoidal rule is the *Adams-Bashforth* method of second order. It is defined by:

$$x(t+h) = x(t) + c_0\dot{x}(t) + c_{-1}\dot{x}(t-h) \quad (8)$$

with

$$\begin{aligned} c_0 &= 3\frac{h}{2} \\ c_{-1} &= -\frac{h}{2} \end{aligned} \quad (9)$$

It's local truncation error is:

$$\varepsilon_l = x(t+h) - x(t) - c_0\dot{x}(t) - c_{-1}\dot{x}(t-h) \quad (10)$$

Using a Taylor approximation,  $\varepsilon_l$  can be approximated by:

$$\varepsilon_l \approx 5\frac{h^3}{12}\ddot{x}(t) \quad (11)$$

Equation (11) shows that the Adams-Bashforth method is of the same order as the trapezoidal rule.

Instead of evaluating directly (7), the difference of predictor and the actual calculated solution can now be used for approximating  $\varepsilon_l$ :

$$\hat{\varepsilon}_l = k(x_P - x) \quad (12)$$

with:

$$\begin{aligned} x_P(t+h) - x(t+h) &= b_1\dot{x}(t+h) + \\ &(b_0 - c_0)\dot{x}(t) - c_{-1}\dot{x}(t-h) \end{aligned} \quad (13)$$

The difference of predictor and corrector is equal to the difference of their local truncation errors (5)-(10):

$$x_P(t+h) - x(t+h) = \varepsilon_{lP} - \varepsilon_{lC} \approx \frac{h^3}{2}\ddot{x}(t) \quad (14)$$

The comparison of (14) with (7) shows that a good approximation for the local truncation error of the trapezoidal rule can be obtained with

$$k = \frac{1}{6}$$

In case of pure sinusoidal wave forms: the local truncation error can be estimated as follows:

$$x(t) = x_0 \cos(\omega t + \varphi_0)$$

$$|\varepsilon_l| \approx \frac{h^3}{12}\ddot{x}(t) = \frac{h^3}{12}\omega_n^3 x_0 \sin(\omega t + \varphi_0)$$

The maximum integration step size, even in a perfect, harmonics free, steady state of the system would be

$$h < \sqrt[3]{\frac{12\varepsilon_{max}}{x_0} \frac{1}{\omega}} \quad (15)$$

if  $\varepsilon_{max}$  was the maximum allowed truncation error.

#### 4.2 Adaptation to Periodic Steady State

The analysis of the local truncation error has shown that the error introduced by discretizing a continuous time system is highly frequency dependant.

The trapezoidal rule works perfect in case of constant signals, and still very good for low frequencies.

But if the steady state of a system is characterized by sinusoidal state variables, the maximum integration step size is very limited, even in steady state.

Hence, an integration method is required having a minimum error around the steady state frequency.

In [3], an integration method has been introduced complying with this requirement. The method is defined by setting in (4):

$$b_0 = b_1 = \frac{1}{\omega_n} \tan\left(\frac{\omega_n h}{2}\right) \quad (16)$$

Unhappily, error estimation is here much more difficult than for classical integration methods because the local truncation error cannot be approximated by a Taylor series any more.

But an adaptive step size algorithm can only be implemented if an error estimation is possible. A precise analysis of the local truncation error according to (5) in the frequency domain is therefore required:

$$\underline{\varepsilon}_l(\omega) = \underbrace{(e^{\omega h} - 1 - b_0 j\omega - b_1 j\omega e^{j\omega h})}_{\underline{\varepsilon}_l(\omega)} \underline{x}(j\omega) \quad (17)$$

Equation (17) can be seen as a linear system with the transfer function  $\underline{\varepsilon}_l(\omega)$  which is called *local error function*, the input  $\underline{x}(\omega)$  and  $\underline{\varepsilon}_l(\omega)$  as output.

The local error function of the normal trapezoidal rule with  $b_0 = b_1 = h/2$  has a triple zero around  $\omega = 0$  and can hence be called a *Butterworth Method* (see Fig. 2).

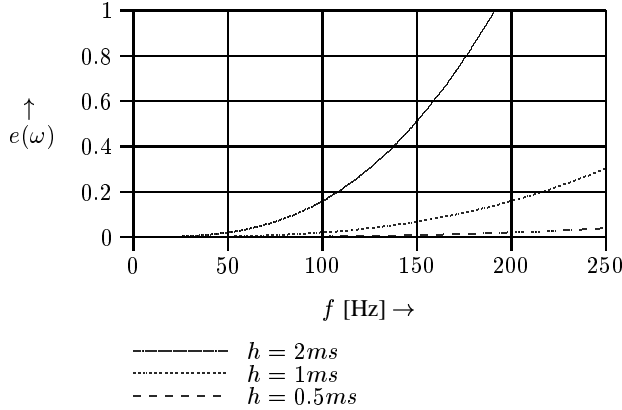


Figure 2: Magnitude of the local error function of the trapezoidal rule

In contrast to *Butterworth methods*, the modified trapezoidal rule according to (16) has zeros at  $\omega = 0$  and  $\omega = \pm\omega_n$ , independent of the integration step size (see fig. 3).

Hence, this method is perfectly adapted to the steady state of instantaneous values of voltages and currents. Because of the zero at  $\omega = 0$ , also slowly decaying DC components are analyzed with high precision even in case of large integration step sizes.

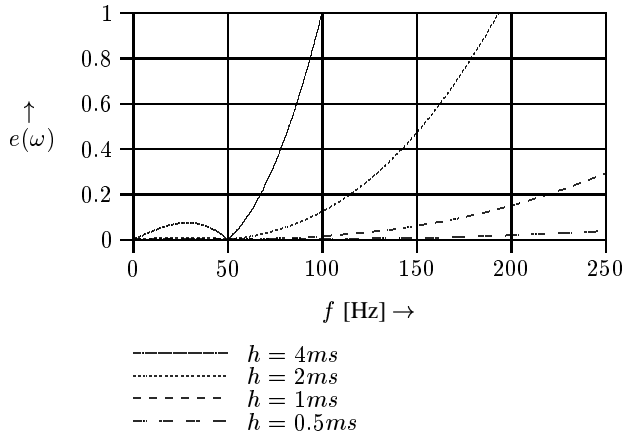


Figure 3: Magnitude of local error function of modified trapezoidal rule

### 4.3 Predictor-Corrector Scheme

The local error of the classical trapezoidal rule could be approximated with the help of a predictor.

Hence, for estimating  $\varepsilon_l$  of the modified trapezoidal rule, a predictor scheme must be found having a local error function similar to  $\underline{\varepsilon}_l(\omega)$  according to (17).

This leads to a *modified Adams-Bashforth method* whose zeros have to be placed at  $\omega = 0$  and  $\omega = \pm\omega_n$  as well.

Transforming (10) into frequency domain leads to the following conditions for the coefficients  $c_0$  and  $c_1$ :

$$\begin{aligned} \underline{\varepsilon}(\omega_n) &= e^{j\omega_n h} - 1 - c_0 j\omega_n - c_{-1} j\omega_n e^{-j\omega_n h} = 0 \\ \underline{\varepsilon}(-\omega_n) &= e^{-j\omega_n h} - 1 + c_0 j\omega_n + c_{-1} j\omega_n e^{j\omega_n h} = 0 \end{aligned} \quad (18)$$

The resulting coefficients are:

$$\begin{aligned} c_{-1} &= -\frac{1 - \cos(\omega_n h)}{\omega_n \sin(\omega_n h)} = -\frac{1}{\omega_n} \tan\left(\frac{\omega_n h}{2}\right) \\ c_0 &= \frac{\cos(\omega_n h) - \cos(2\omega_n h)}{\omega_n \sin(\omega_n h)} \end{aligned} \quad (19)$$

The local truncation error can now be approximated by the difference of a predictor solution  $x_P(t+h)$  obtained with the modified Adams Bashforth method according to (19) and the actual solution  $x(t+h)$  calculated by the modified trapezoidal rule.

$$\hat{\varepsilon}(\omega) = k |\underline{x}_P - \underline{x}| = k |(\underline{\varepsilon}_P(\omega) - \underline{\varepsilon}(\omega))| \quad (20)$$

Figure 4 shows the magnitudes of the exact and the approximated local error function.

It can be seen that the approximated error is a very good estimate for the local error function, at least in the frequency band of interest. If necessary, it could further be improved by adjusting the factor  $k$  in (20).

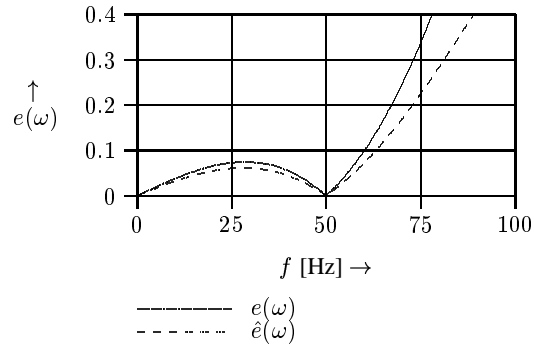


Figure 4: Estimated and exact local error function of modified trapezoidal rule ( $h = 4ms, k = 1/6$ )

### 4.4 Numerical Oscillations

Another requirement for the numerical integration method is A-stability. Only together with an A-stable method, the integration step size can be increased independent of the eigenvalues of the electrical network.

The easiest way of analyzing the stability of implicit, linear integration methods is their description as a mapping of complex frequencies.

Methods according to (4) are performing the following mapping of complex frequencies:

$$z = \frac{1 + b_0 p}{1 - b_1 p} \quad (21)$$

The condition for A-stability of methods according to (4) is (e.g. [3]):

$$b_1 \geq b_0 \quad (22)$$

The modified trapezoidal rule is therefore A-stable if  $h$  lies in one of the following ranges

$$\omega_n \frac{h}{2} \in \left( k\pi, k\pi + \frac{\pi}{2} \right) \equiv h \in \left( k \frac{1}{f_n}, \left( k + \frac{1}{2} \right) \frac{1}{f_n} \right) \quad (\text{k integer}) \quad (23)$$

Numerical oscillations occur if real eigenvalue  $p_i$  of the original system are mapped on oscillatory eigenvalues  $z_i$  of the corresponding discrete time system.

These numerical oscillations can always occur if integration methods according to (4) are used. The important thing is, that these numerical oscillations are well damped so that they are decaying quickly whenever they occur.

The sensitivity of an integration method to numerical oscillations can be analyzed using (21) by:

$$\lim_{\sigma \rightarrow -\infty} \frac{1 + b_0(\sigma + j\omega)}{1 - b_1(\sigma + j\omega)} = -\frac{b_0}{b_1} \quad (24)$$

In case of integration methods with  $b_0 = b_1 = b$ :

$$\lim_{\sigma \rightarrow -\infty} \frac{1 + b(\sigma + j\omega)}{1 - b(\sigma + j\omega)} = -1 \quad (25)$$

Very large eigenvalues are therefore mapped near to  $z = -1$ . Using

$$-1 = e^{j\pi}$$

it can be seen that  $z = -1$  corresponds to an oscillatory mode at  $f = \frac{1}{2h}$ .

For the practical use of the modified integration method, it is absolutely necessary that these numerical oscillations are well damped which requires:

$$\frac{b_0}{b_1} < 1$$

In order to comply with this condition, a damped modified trapezoidal rule can be defined by:

$$\begin{aligned} b_1 &= \frac{1}{\omega_n} \tan \left( \left( 1 - \frac{\alpha}{2} \right) \omega_n h \right) \\ b_0 &= \frac{1}{\omega_n} \tan \left( \frac{\alpha}{2} \omega_n h \right) \end{aligned} \quad (26)$$

Equation (26) complies with the condition  $b_1 > b_0$  for  $0 < \alpha < 1$  and is hence damping numerical oscillations.

In order to match the precision requirements at  $\omega = \omega_n$ ,  $\alpha$  has to be placed very near to 1. The local error function is then not equal zero at nominal frequency but it has still a very low value.

For demonstrating some applications of the described methods, an unbalanced fault is simulated in a power system similar to the IEEE 9-bus bar system [7] with various algorithms.

All machines are voltage controlled but no governor models have been connected in order to obtain considerable speed deviations.

The machine models are of eight order (including zero sequence). Lines are modelled by several  $\Pi$ -elements connected in series.

The simulated event is a double phase to ground fault on line 5-7 at 50% distance with a duration of 300ms. The fault is cleared by switching off the faulted line.

### 5.1 Multirate Algorithm

Annex 1 shows voltage phasor and speed for the simulated event. The step sizes are  $h_f = 0.05ms$  and  $h_s = 10ms$ . Because the anti-alias filter cannot damp perfectly the negative sequence component, the electrical torque is subsampled. However, according to large time scale approximations, only the average value of speed is of interest. If a higher precision is required the ratio  $h_s/h_f$  has to be decreased.

But the influence of these effects to voltage is only minor as the magnitude of the voltage phasor in fig. 6 shows clearly.

### 5.2 Adaptive Step Size Algorithm

The same fault has been simulated using the adaptive step size algorithm. Annex 2 shows that the simulation with step size adaptation is in great accordance to a normal EMT-simulation.

Annex 3 shows results of the same simulation run but in a larger time scale.

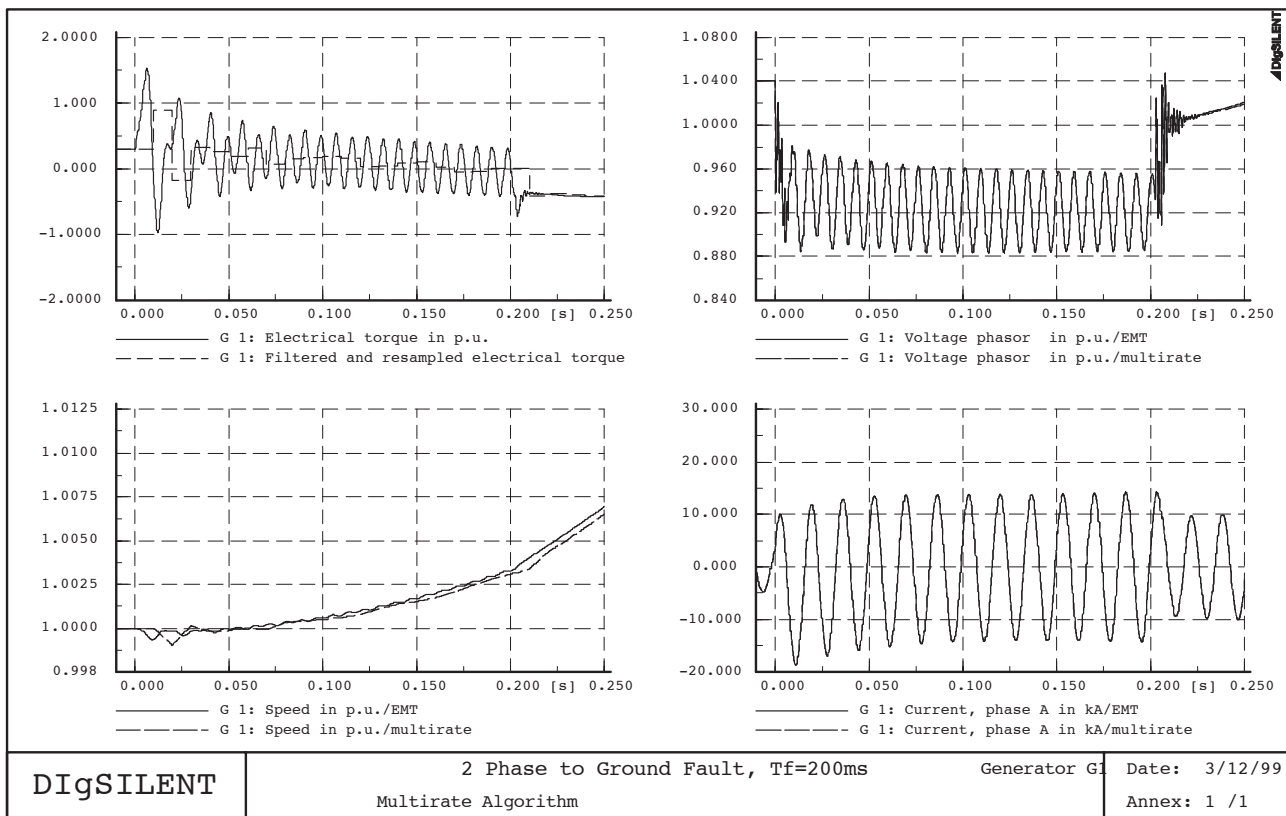
Even if there is a considerable speed deviation and hence a detuning of the integration method, the integration step size can be increased up to  $h = 6.4ms$ , without any considerable drawback to the precision of the numerically calculated state variables.

## CONCLUSION

Two methods have been presented which are increasing the performance of simulation programs for power systems considerably.

The multirate method uses different integration step sizes for different state variables. It can be applied for linear and nonlinear systems under the condition that the state variables of the slow subsystem are only weakly influenced by fast transients. The described decomposition is leading to such a form in almost all cases.

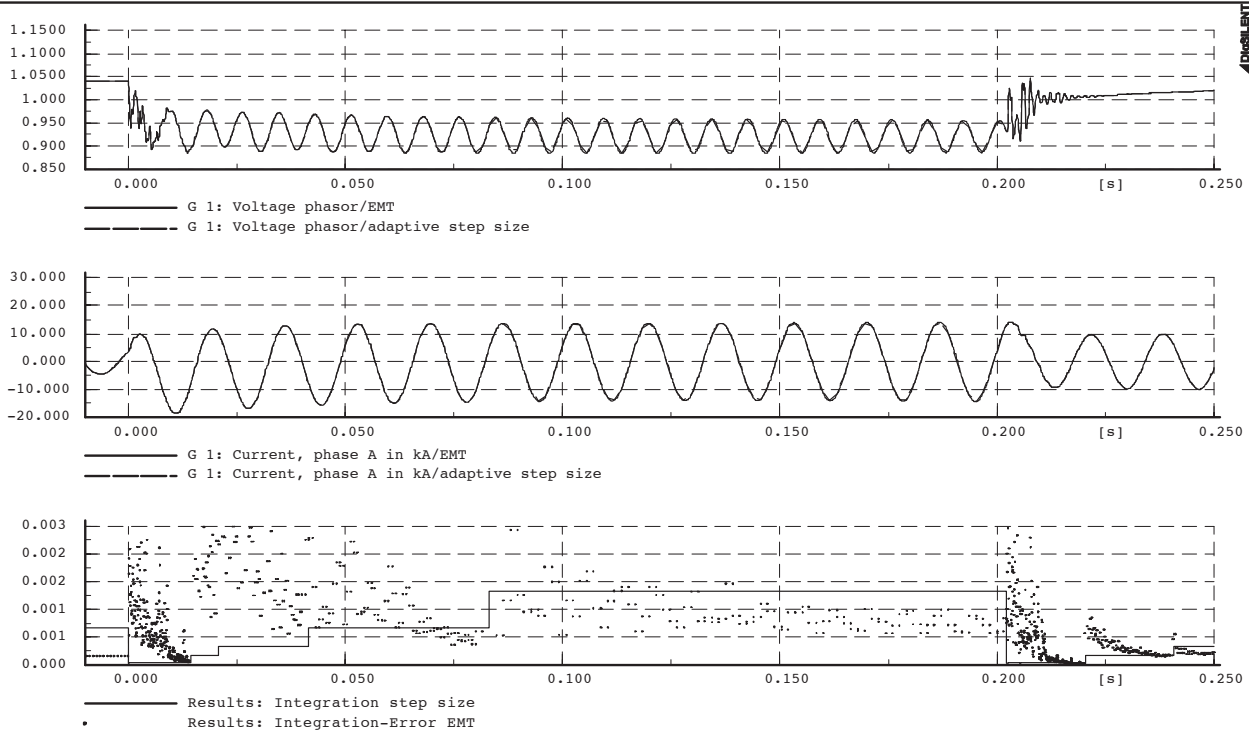
The second method adapts the integration step size in time. By modifying the trapezoidal rule, the integration step size can be increased up to  $h = 10ms$  if voltages and currents are reaching their sinusoidal steady state. Limitations are here systems with a considerable high amount of harmonics



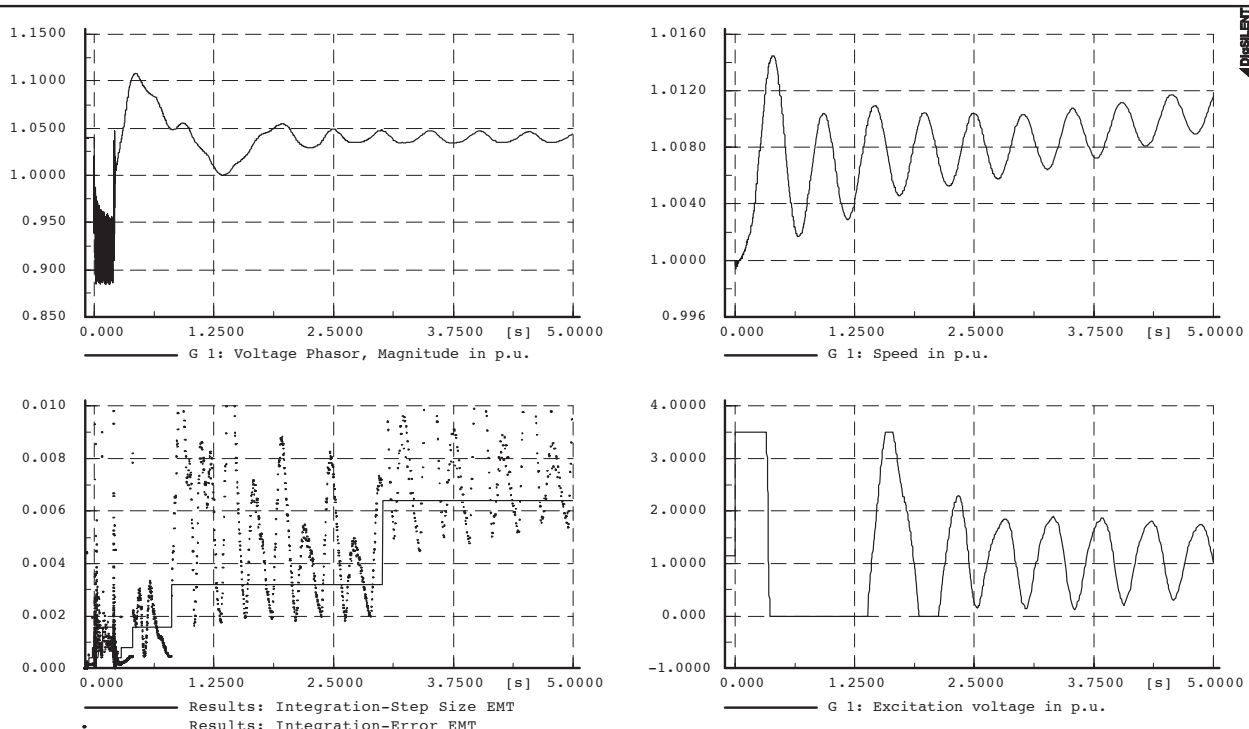
which are limiting the maximum integration step size. The adaptive step size method is particularly well suited for applications in which mutual influences between the electrical grid and the electromechanical system have to be taken into account. Subsynchronous resonances or self excitation of induction machines belong to these applications. Here, a steady state grid model cannot be used and traditional EMT-programs are leading to huge calculation times even if there are only slow transients to be analyzed.

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DigSILENT	2 Phase to Ground Fault, Tf=200ms	Generator G1	Date: 3/12/99
	Adaptive step size algorithm		Annex: 2 / 1



DigSILENT	2 Phase to Ground Fault Tf=200ms	Electromechanical Transient	Date: 3/12/99
	Adaptive step size algorithm		Annex: 3 / 1